

Natural Convection Heat Transfer of Cu-water Nanofluid within a Square Enclosure: A Numerical Investigation

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Abstract—Natural convection heat transfer and fluid flow within a square enclosure filled up with Cu-water nanofluid has been investigated numerically exploiting finite volume method. The system of conservation equations representing of continuity, momentum and energy in dimensionless form are solved by using finite volume SIMPLEC algorithm. The upper and lower walls of the cavity are well insulated and impermeable. However, the lateral walls are imposed to constant temperatures. The Maxwell-Garnett model is used to predict the ratio of thermal conductivity. Results are posed for different values of Rayleigh number and volume fraction, in terms of streamlines, isotherms and local Nusselt number. The effects of nanoparticle volume fraction are debated on heat transfer characteristic inside the cavity.

1. INTRODUCTION

Enhancement of heat transfer inside enclosures is indispensable from energy and industrial perspectives. Furthermore, extensive investigations have been carried out for analysis of heat transfer and fluid flow phenomena occurring within enclosures especially square one, for its wide engineering applications. Low thermal conductivity of conventional working fluids, namely water and oil is so severe a limitation for performance and size compactness in these equipment. An innovative technique to increase the efficiency of these systems is to utilize nanofluids. Nanofluids belong to a class of fluid comprised of nano-sized solid particles (nanoparticles), suspended in a pure base fluid. Suspension of these nanoparticles has been realized to enhance the thermal conductivity of the base fluid. As a result of appreciably higher thermal conductivity of nanofluids, the overall heat transfer within the enclosures augments and this has been reported by numerous experiments conducted by various researchers over the prior decade. Choi [1] was the pioneer researcher who used the term nanofluids to refer to the fluid with suspended nanoparticles. His research illustrated that the addition of nanoparticles to pure base fluid enhances the thermal conductivity of the fluid up to approximately two times. Khanafer et al. [2] studied buoyancy-driven convection in a rectangular enclosure filled up with nanofluid. They found that heat transfer across the enclosure augments with the

volumetric fraction of copper nanoparticles (Cu) for different Grashof numbers. Oztop et al. [3] conducted a numerical study on flow field and temperature distributions in a partially heated square enclosure filled up with nanofluid. Therefore, they concluded that the type of nanoparticle plays a pivotal factor for heat transfer enhancement. Mahmoodi et al. [4] studied natural convection of Cu-water nanofluid inside a square cavity with an adiabatic square body located in its center. Their results anticipated that when the volume fraction of the nanoparticles is kept constant, the rate of heat transfer increases by the surge of Rayleigh number.

2. MATHEMATICAL MODELLING

2.1 Governing equations

A square enclosure with length of L , considered in this study filled up with a Cu nanoparticles is shown in Fig. 1. While left and right walls are maintained at constant temperatures of T_H and T_C respectively, both top and bottom walls are assumed to be adiabatic and impermeable.

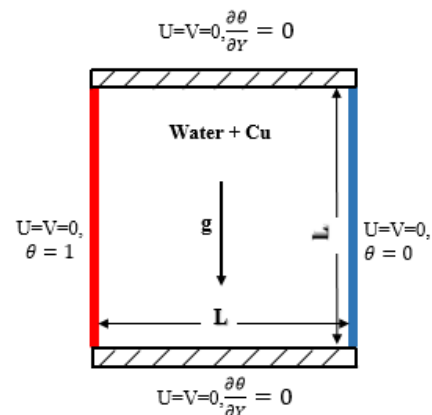


Fig. 1: A schematic view of the discussed square filled up with Cu nanoparticles and its governing boundary conditions

The nanofluid is Newtonian, incompressible and laminar. The pure base fluid and nanoparticles are assumed to be in thermal equilibrium state and no slip occurs between them. The nanoparticles are deemed to have uniform shape (spherical). Moreover, in comparison with other modes of heat transfer, radiation heat transfer between the hot and cold lateral walls of the cavity is negligible.

Therefore, with the above assumptions the steady state two dimensional Cartesian equations of continuity, momentum and energy conservation in dimensionless form will be as follows [5]:

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

X-Momentum:

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr^* \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (2)$$

Y-Momentum:

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr^* \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra^* \times Pr^* \times \theta \quad (3)$$

Energy:

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (4)$$

Where the dimensionless variables are introduced as:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha}, V = \frac{vL}{\alpha}, P = \frac{pL^2}{\rho\alpha^2} \quad (5)$$

$$\theta = \frac{T - T_c}{T_H - T_c}, Pr = \frac{\nu}{\alpha}, Pr^* = \frac{\mu_{eff} c_{p, nf} k_f}{\mu_f c_{p, f} k_{eff}}$$

$$Ra = \frac{g\beta(T_H - T_c)L^3}{\nu\alpha} \text{ and}$$

$$Ra = \frac{(\rho\beta)_{nf} k_f (\rho c_p)_{nf} \mu_f}{(\rho\beta)_f k_{eff} (\rho c_p)_f \mu_{eff}} Ra$$

Ra^* and Pr^* are Rayleigh and Prandtl numbers for nanofluid, respectively, and are pertinent to the Rayleigh and Prandtl numbers assessed for pure base fluid. The local Nusselt number, (Nu_l) , along the hot lateral wall of the enclosure is expressed as:

$$Nu_l = -\frac{h \times L}{k_f} \quad (6)$$

Therefore, the average Nusselt number of the hot lateral wall can be acquired by integrating the local Nusselt number described in Eq. (6) as below:

$$\overline{Nu} = -\frac{1}{L} \int_0^1 \frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial X} \Big|_{Hot\ wall} dY \quad (7)$$

2.2 Nanofluid properties

The physical properties of the nanofluid in the above equations are as follows [6]:

Density:

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_p \quad (8)$$

Heat capacitance:

$$(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_p \quad (9)$$

Volumetric expansion coefficient:

$$(\rho\beta)_{nf} = (1 - \varphi)(\rho\beta)_f + \varphi(\rho\beta)_p \quad (10)$$

Dynamic viscosity:

$$\mu_{eff} = \frac{\mu_f}{(1 - \varphi)^{2.5}} \quad (11)$$

Thermal conductivity:

$$\frac{k_{eff}}{k_f} = \frac{k_p + (n - 1)k_f - \varphi(n - 1)(k_f - k_p)}{k_p + (n - 1)k_f + \varphi(k_f - k_p)} \quad (12)$$

Thermal diffusivity:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (13)$$

The thermo-physical properties of water were obtained from ASHRAE Handbook [7] which was curve fitted as a function of temperature with the below equations. The thermo-physical properties of water and Cu nanoparticles at 300K are tabulated in table 1, as well.

$$\rho_f = -0.0045T^2 + 2.45T + 667.83 \quad (14)$$

$$k_f = -8.2 \times 10^{-6}T^2 + 0.00636T - 0.5552 \quad (15)$$

$$\mu_f = 0.796 \times \exp(-0.0227 \times T) \quad (16)$$

Table 1: Thermo-physical properties of pure base fluid and solid nanoparticles. [8]

Physical Properties	Pure base fluid (Water)	Nanoparticle (Cu)
C_p (j/kgK)	4179	385
ρ (kg/m ³)	997.1	8933
k (w/mK)	0.613	400
β (1/K)	21×10^{-5}	1.67×10^{-5}
μ (N.s/m ²)	8.91×10^{-4}	—

3. NUMERICAL APPROACH AND VALIDATION

The system of governing equations was solved by the control volume method exploiting FLUENT software. Discretization of convection terms and other quantities resulting from the governing equations was conducted using first-order upwind scheme. The pressure-velocity coupling was handled by SIMPLEC algorithm. For all the simulations performed in this paper, the solutions were fully converged since the residuals were smaller than 10^{-6} .

3.1 Grid Independency

To examine and assess grid-dependency test, the present numerical code was performed as represented in figures 2-4.

To capture the swift variations in the dependent variables namely temperatures, horizontal and vertical velocities within the square enclosure, 4 different non-uniform square grids were implemented. As it is depicted, the grid mesh of 90×90 is adequate to describe the heat transfer and fluid flow phenomena inside the cavity. Furthermore, it is evident that further increment in the grids results in the same outcome.

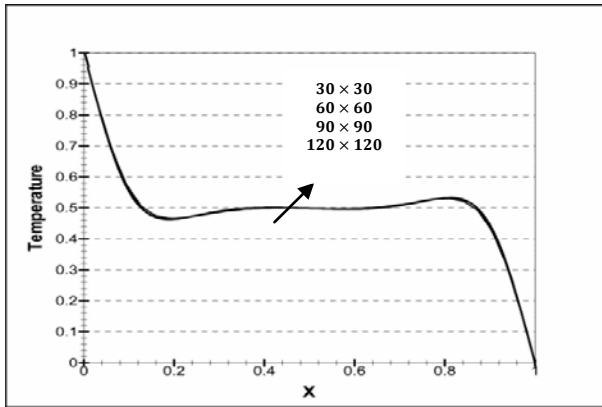


Fig. 2. Temperature profiles, θ , at the mid-section of the cavity for various mesh sizes ($Ra = 10^6, \phi = 0.04$).

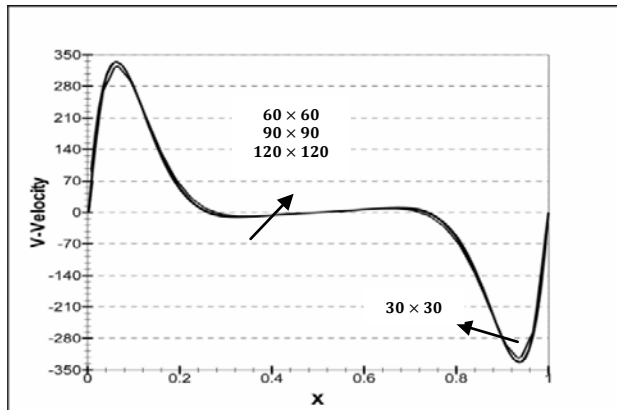


Fig. 3. Vertical velocity, V , profiles at the mid-section of the cavity for various mesh sizes ($Ra = 10^6, \phi = 0.04$).

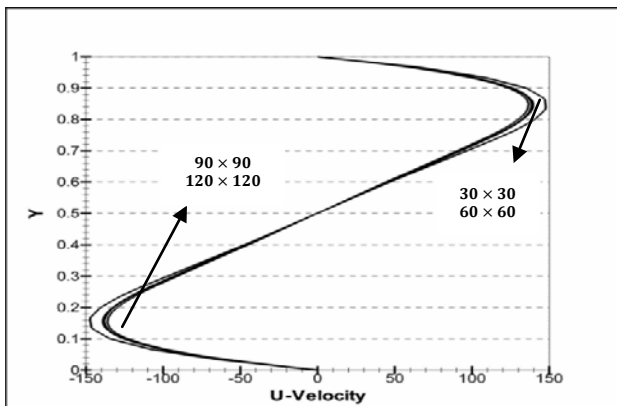


Fig. 4. Horizontal velocity, U , profiles at the mid-section of the cavity for various mesh sizes ($Ra = 10^6, \phi = 0.04$).

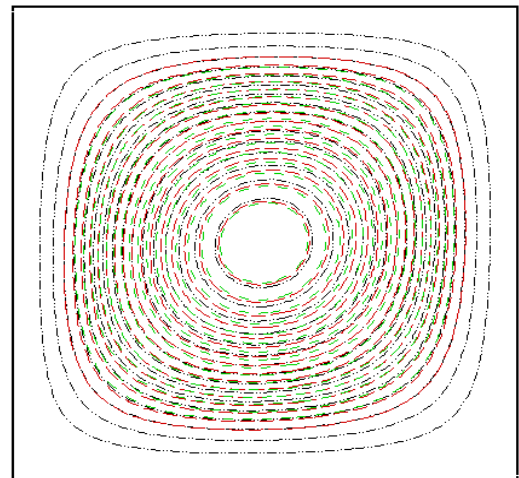
Table 2: Comparison of pure fluid solution with previous works for different Rayleigh numbers. [9]

	Present	Barakos Mitsoulis	De Vahl Davis	Markatos Pericleous
$Ra = 10^4$ \overline{Nu}	2.193	2.245	2.243	2.201
$Ra = 10^5$ \overline{Nu}	4.504	4.510	4.519	4.43
$Ra = 10^6$ \overline{Nu}	8.776	8.806	8.799	8.754

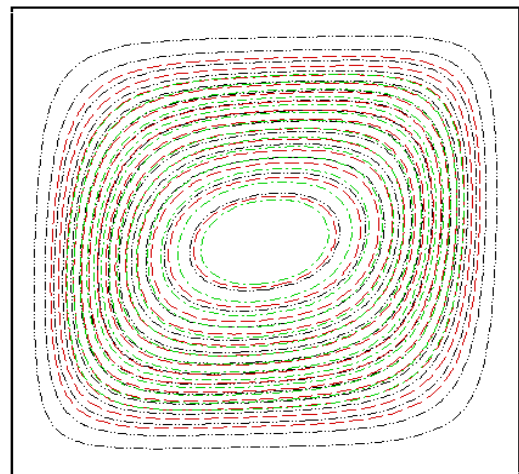
3.2 Validation

To validate the present code, the results of average Nusselt number along the hot wall compared with data published by o-

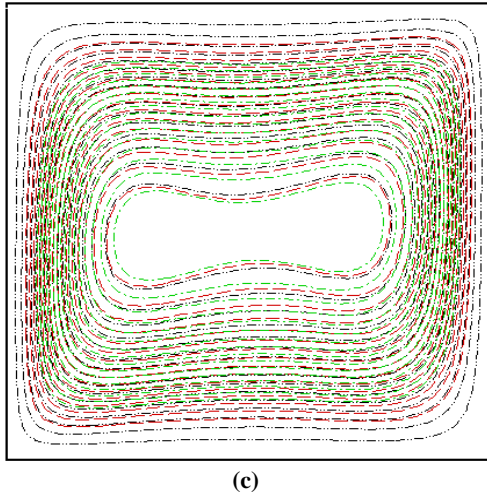
Streamlines



(a)

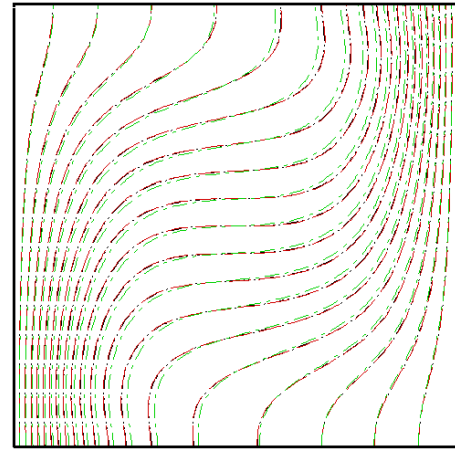


(b)

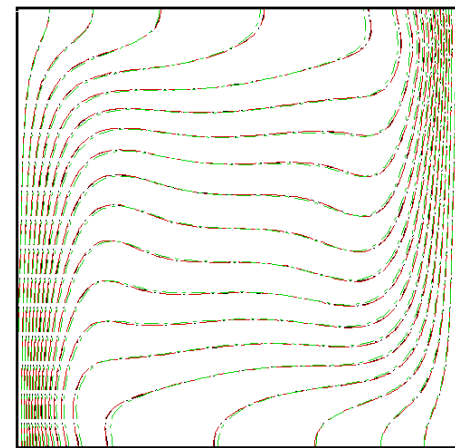


(c)

Fig. 5: Streamlines for $\phi = 0$ (Dash-Dot-Dot lines), $\phi = 0.04$ (Long-Dash lines) and $\phi = 0.08$ (Dashed lines), (a) $Ra = 10^4$, (b) $Ra = 10^5$, (c) $Ra = 10^6$



(b)



(a)

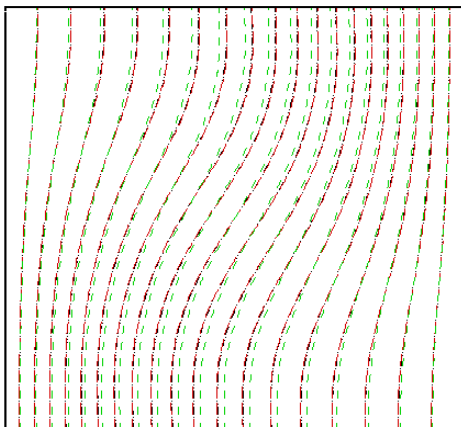
ther researchers since pure workingfluid was used in the cavity for different Rayleighnumbers and $Pr = 0.7$. As it can be observed, table 2 illustrates an excellent agreement between the present results and other benchmark solutions.

4. RESULTS AND DISCUSSIONS

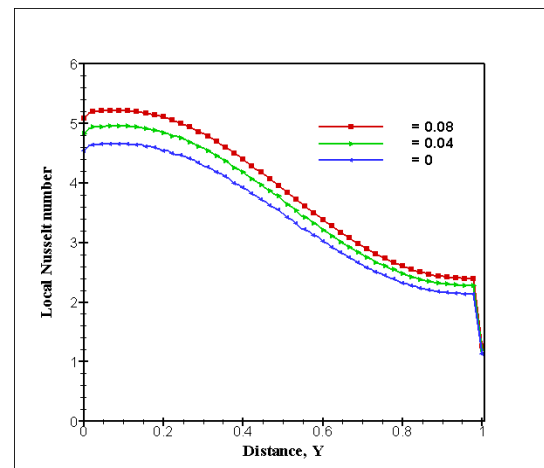
A numerical analysis has been conducted to investigate the impacts of nanoparticles volume fraction and Rayleigh number on a Cu-water nanofluid. The enclosure is differentially heated and the other walls are adiabatic. Assessments were made for various values of volume fraction ($0 < \phi < 0.08$), and Rayleigh numbers ($10^4 < Ra < 10^6$). Figures 5 (a)-(c) and 6 (a)-(c), show the effects of Rayleigh number on streamlines (on the left) and isotherms (on the right) for different volume fraction of nanoparticles. The dash-dot-dot lines represent the condition in which no nanoparticle is used inside the base fluid ($\phi = 0$), while the l-

Fig. 6: Isotherms for $\phi = 0$ (Dash-Dot-Dot lines), $\phi = 0.04$ (Long-Dash lines) and $\phi = 0.08$ (Dashed lines), (a) $Ra = 10^4$, (b) $Ra = 10^5$, (c) $Ra = 10^6$

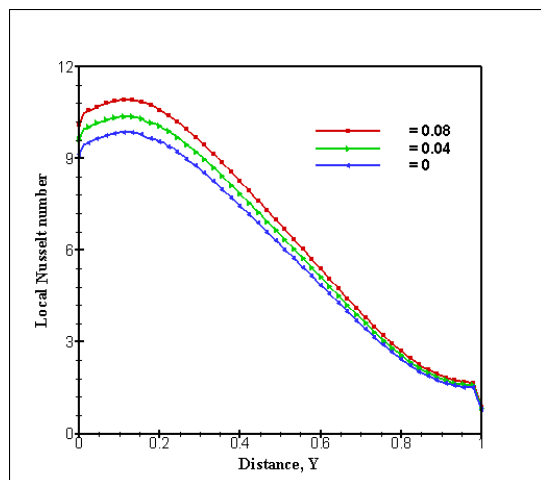
Isotherms



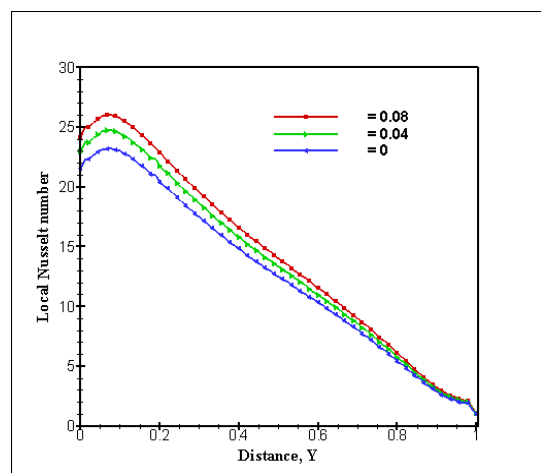
(a)



(a)



(b)



(c)

Fig. 7: Local Nusselt number along the heated wall for different Cu nanoparticle volume fraction, (a) $Ra = 10^4$, (b) $Ra = 10^5$, (c) $Ra = 10^6$

ong-dash lines present the condition since the nanoparticle volume fraction is ($\phi = 0.04$). Furthermore, dashed-lines are applied to specify the condition in which ($\phi = 0.08$). The results evidently show that using a nanofluid would decline the strength of flow in the cavity as the maximum value of the stream function is 1.16 since no nanoparticles are used. 5.23 when the copper nanoparticles volume fraction is 0.04, and 11.21 when the volume fraction of nanoparticle is 0.08, as well. In comparison with the low conductivity coefficient of the pure base fluid, the conductivity coefficient of nanoparticles is higher. Thus logically, by utilizing nanofluids, heat transfer characteristics in the enclosure can be enhanced. Additionally, it can be deduced from the featured figures that since the volume fraction of nanoparticles is increased, the thickness of thermal boundary layers near the active walls thins. In another words, for larger values of volume fraction of nanoparticles, heat transfer rate will be augmented.

Fig. 7 (a)-(c) present the fluctuation of local Nusselt number on the heated wall of enclosure for different Rayleigh number ($10^4 < Ra < 10^6$) since Cu nano-particles are used. Moreover, local Nusselt number is plotted for different values of nanoparticle volume fraction. As it can be comprehended, the local Nusselt number enhances when the volume fraction of Cu nanoparticles is increased. The main reason is the high thermal conductivity of copper nanoparticles, which impact on conductive heat transfer mechanism. In the point of fact, in contrast to pure base fluid, some of flow strength in the cavity will be lost as Cu nanoparticles are utilized. Furthermore, it is also inferred that the local Nusselt number augments substantially since the Rayleigh number increases from 10^4 to 10^6 by comparing these figures.

5. CONCLUSION

A numerical study has been conducted to investigate precisely, the effect of using nanofluid on natural convection in a square cavity. Some of the pivotal points of the attained results of the present code can be drawn as the following:

- Since Cu nanoparticles are used, heat transfer rate from the active wall of the cavity is augmented by increasing the value of Rayleigh number and nanoparticle volume fraction.
- Enhancement in conductive heat transfer mechanism is realized due to higher thermal conductivity of Cu nanoparticles, compared with pure base fluid.
- Thinning the thickness of thermal boundary layers near the active walls, the rate of heat transfer increases.
- By increasing the values of nanoparticle volume fraction the flow strength will be diminished in the cavity.

6. NOMENCLATURES

C_p	Specific heat, $Jkg^{-1}k^{-1}$
g	Gravity acceleration, m/s^2
Nu	Local Nusselt number
P	Dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
T	Temperature, K
T_c	Temperature on the cold wall, K
T_h	Temperature on the hot wall, K
u	x component of velocity, ms^{-1}
U	x component of dimensionless velocity
v	y component of velocity, ms^{-1}
V	y component of dimensionless velocity

Greek Letters

θ	Dimensionless temperature
ρ	Density, kgm^{-3}
ϕ	Nanoparticle volume fraction
β	Volumetric coefficient of thermal expansion, K^{-1}
μ	Dynamic viscosity, $kgm^{-1}s^{-1}$

Subscript

eff	effective
f	fluid
nf	nanofluid
p	nanoparticle

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